

1. Coordinate transformations

1.1. 2D

Point A in Figure 1.1 has coordinates x_A and y_A in system xO_0y and X_A and Y_A in system XOY . Coordinates in one system can be calculated as functions of the coordinates in the other system according to relations (1.1) and (1.2).

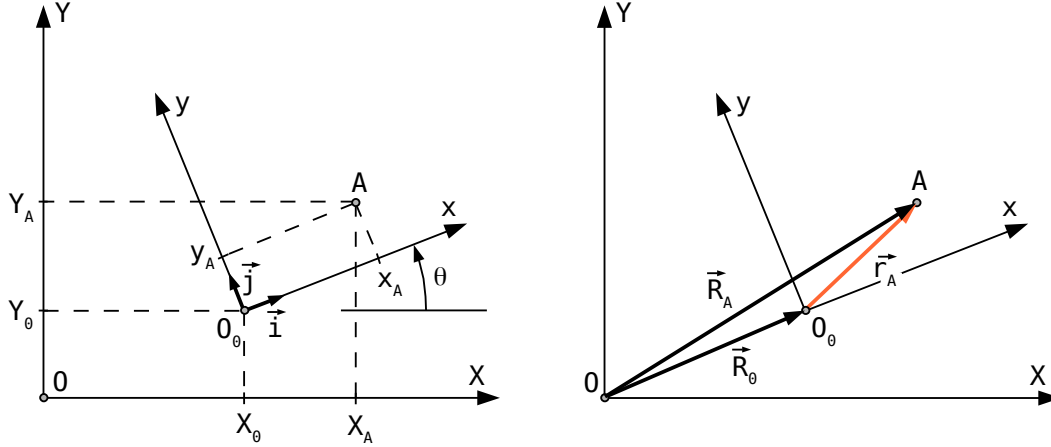


Figure 1.1 - Coordinates of a point in 2 systems of axes

$$\vec{i} = \begin{pmatrix} i_x \\ i_y \end{pmatrix} = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} ; \quad \vec{j} = \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix} ; \quad [C] = \begin{pmatrix} \vec{i} & \vec{j} \end{pmatrix} = \begin{pmatrix} i_x & j_x \\ i_y & j_y \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} ;$$

$$\begin{pmatrix} X_0 \\ Y_0 \end{pmatrix} = \vec{R}_0 ; \quad \begin{pmatrix} X_A \\ Y_A \end{pmatrix} = \vec{R}_A = \vec{R}_0 + \vec{r}_A ; \quad \vec{r}_A = x_A \cdot \vec{i} + y_A \cdot \vec{j} ;$$

$$\begin{pmatrix} X_A \\ Y_A \end{pmatrix} = \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix} + x_A \cdot \begin{pmatrix} i_x \\ i_y \end{pmatrix} + y_A \cdot \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix} + \begin{pmatrix} i_x & j_x \\ i_y & j_y \end{pmatrix} \cdot \begin{pmatrix} x_A \\ y_A \end{pmatrix} = \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix} + [C] \cdot \begin{pmatrix} x_A \\ y_A \end{pmatrix} \quad (1.1)$$

$$\begin{pmatrix} X_A \\ Y_A \end{pmatrix} - \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix} = [C] \cdot \begin{pmatrix} x_A \\ y_A \end{pmatrix} \rightarrow \begin{pmatrix} x_A \\ y_A \end{pmatrix} = [C]^{-1} \cdot \begin{pmatrix} X_A - X_0 \\ Y_A - Y_0 \end{pmatrix} = [C]^T \cdot \begin{pmatrix} X_A - X_0 \\ Y_A - Y_0 \end{pmatrix} \quad (1.2)$$

where i_x , i_y , j_x and j_y are projections of unit vectors \vec{i} and \vec{j} of axes x and y on axes X and Y .

1.2. 3D

$$[C] = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \end{pmatrix} = \begin{pmatrix} i_x & j_x & k_x \\ i_y & j_y & k_y \\ i_z & j_z & k_z \end{pmatrix} ; \quad \begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix} = \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} + [C] \cdot \begin{pmatrix} x_A \\ y_A \\ z_A \end{pmatrix} \quad (1.3)$$

$$\begin{pmatrix} x_A \\ y_A \\ z_A \end{pmatrix} = [C]^T \cdot \begin{pmatrix} X_A - X_0 \\ Y_A - Y_0 \\ Z_A - Z_0 \end{pmatrix} \tag{1.4}$$